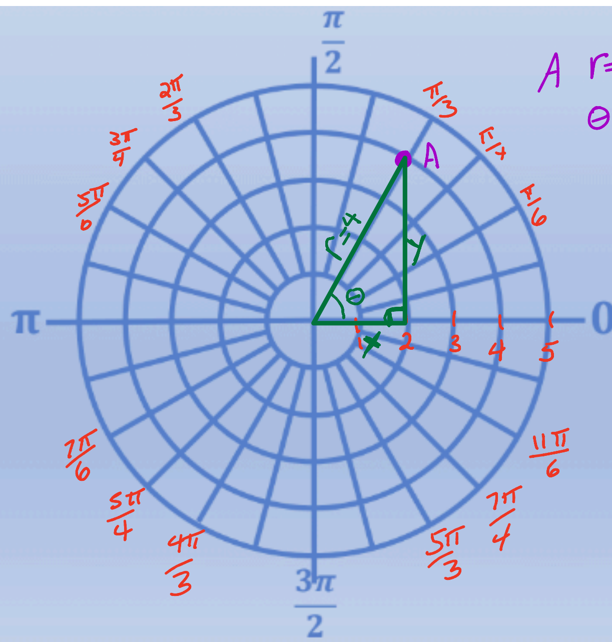
Period 3, May 28, 2025

$\frac{\pi}{3}+2\pi=\frac{\pi}{3}+\frac{6\pi}{3}=\frac{7\pi}{3}$



A $r=4$

$\theta=\frac{\pi}{3}$

$\left(4,\frac{\pi}{3}\right), \left(4,\frac{7\pi}{3}\right), \left(-4,\frac{4\pi}{3}\right), \left(-4,-\frac{2\pi}{3}\right)$

$r^2=x^2+y^2$

$r=\sqrt{x^2+y^2}$

$r\cdot\sin\theta=\frac{y}{\cancel{r}}\cdot\cancel{r}$

$r\cdot\cos\theta=\frac{x}{\cancel{r}}\cdot\cancel{r}$

$r\sin\theta=y$

$r\cos\theta=x$

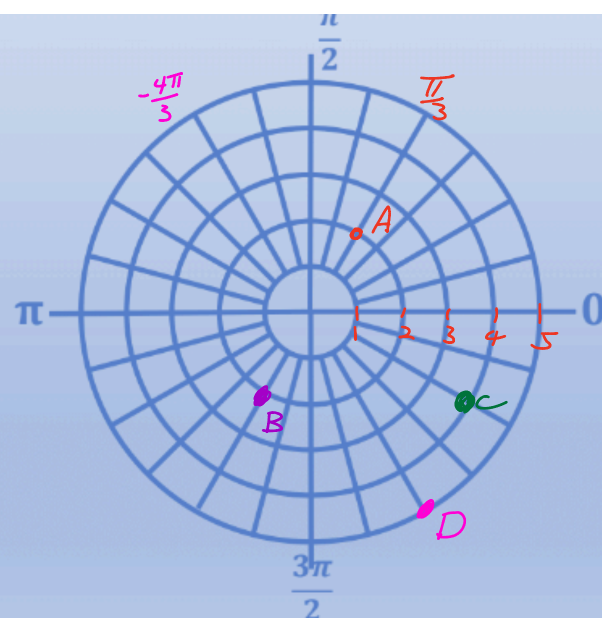
$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{y}{r}}{\frac{x}{r}}=\frac{y}{\cancel{r}}\cdot\frac{\cancel{r}}{x}=\frac{y}{x}$

Graph the following points:

$\left(2,\frac{\pi}{3}\right)$ A $\quad \left(-2,\frac{\pi}{3}\right)$ B

$\left(4,-\frac{\pi}{6}\right)$ C $\quad \left(4,\frac{11\pi}{6}\right)$

$\left(-5,\frac{-4\pi}{3}\right)$ D $\quad \left(-3,\frac{7\pi}{3}\right)$



Ex. Change from polar to rectangular

$(-4, \pi)$ r θ

$x = -4\cos\pi = -4\cdot -1 = 4$

$y = -4\sin\pi = -4\cdot 0 = 0$

$(4, 0)$

$\left(\sqrt{2}, \frac{\pi}{4}\right)$

$x = \sqrt{2}\cdot\cos\frac{\pi}{4} = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{2}{2} = 1$

$y = \sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{2}{2} = 1$

$(1, 1)$

Ex. Change from rectangular to polar (r, θ)

$(1, -\sqrt{3})$

$r^2 = (1)^2 + (-\sqrt{3})^2 = 1 + 3 = 4$

$r^2 = 4 \Rightarrow r = 2$

$\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \Rightarrow \theta = \frac{-\pi}{3}$

$\left(2, \frac{5\pi}{3}\right), \left(2, -\frac{\pi}{3}\right)$

$(0, -2)$

$r^2 = 0^2 + (-2)^2 = 4$

$r = 2$

$\left(2, \frac{3\pi}{2}\right)$

$\tan\theta = \frac{-2}{0} = \phi$

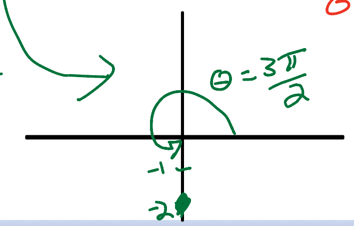
$x = r\cos\theta$

$y = r\sin\theta$

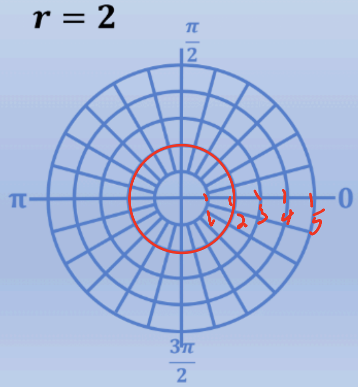
$\tan\theta = \frac{y}{x}$

$r^2 = x^2 + y^2$

Graph Point



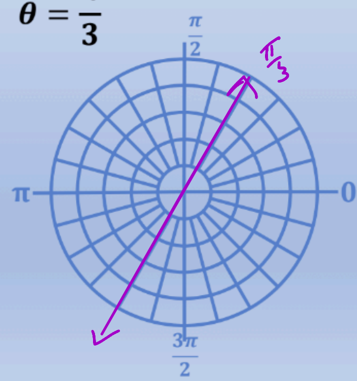
$r = 2$



$2^2 = x^2 + y^2$

$4 = x^2 + y^2$ circle

$\theta = \frac{\pi}{3}$

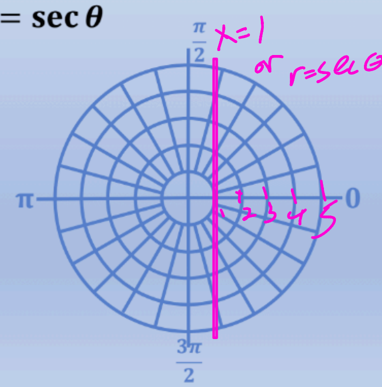


$\tan\frac{\pi}{3} = \frac{y}{x}$

$\sqrt{3} = \frac{y}{x}$

$x = \sqrt{3}\cdot y$

$r = \sec\theta$



$r = \sec\theta$

$\cos\theta\cdot r = \cancel{\frac{1}{\cos\theta}\cdot\cos\theta}$

$r\cos\theta = 1$

$x = 1$

$\boxed{r = 3\cos 5\theta = f(\theta)}$

$r = F(\theta)$

$y = r\sin\theta = f(\theta)\sin\theta \Rightarrow \frac{dy}{d\theta} = F'(\theta)\sin\theta + F(\theta)\cos\theta$

$x = r\cos\theta = F(\theta)\cos\theta \Rightarrow \frac{dx}{d\theta} = F'(\theta)\cos\theta + F(\theta)(-\sin\theta)$

$$\text{Slope} = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{dy}{\cancel{d\theta}} \cdot \frac{\cancel{d\theta}}{dx} = \frac{F'(\theta)\sin\theta + F(\theta)\cos\theta}{F'(\theta)\cos\theta - F(\theta)\sin\theta}$$

Slope

$F(\theta) = 3\cos 5\theta$

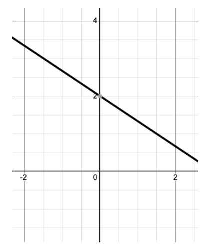
$F'(\theta) = 3(-\sin 5\theta) \cdot 5 = -15\sin 5\theta$

$$\frac{dy}{dx} = \frac{-15\sin 5\theta \cdot \sin\theta + 3\cos 5\theta \cdot \cos\theta}{-15\sin 5\theta\cos\theta - 3\cos 5\theta \cdot \sin\theta} = \text{slope}$$

---

9) $2x + 3y = 6$

$2x + 3y = 6$



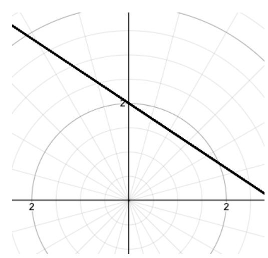
$2 \cdot r\cos\theta + 3r\sin\theta = 6$

$$\frac{r(\cancel{2\cos\theta + 3\sin\theta})}{\cancel{(2\cos\theta + 3\sin\theta)}} = \frac{6}{2\cos\theta + 3\sin\theta}$$

$$r = \frac{6}{2\cos\theta + 3\sin\theta}$$

$$r = \frac{6}{2\cos\theta + 3\sin\theta}$$

$0 \le \theta \le 12\pi$



$r = 8\sin\theta + 3\cos\theta$

$0 \le \theta \le \pi$

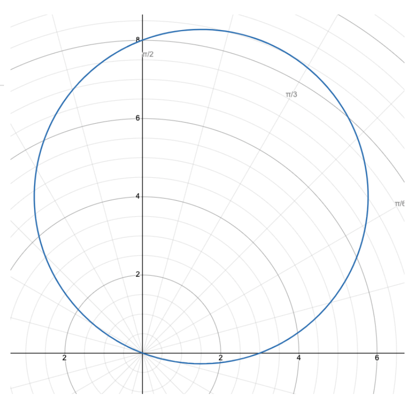
13) $r - \cancel{3\cos\theta} = 8\sin\theta$

$\quad + \cancel{3\cos\theta} \qquad +3\cos\theta$

$r \cdot r = (8\sin\theta + 3\cos\theta) \cdot r$

$r^2 = 8r\sin\theta + 3r\cos\theta$

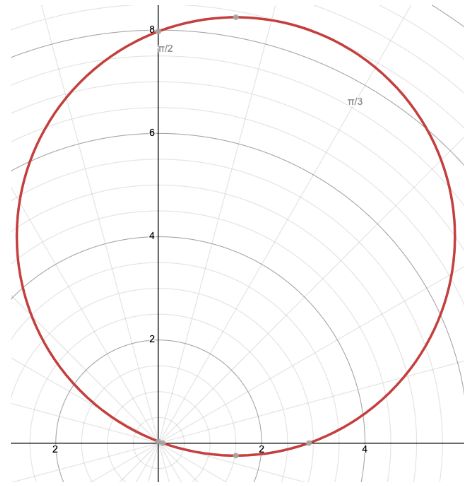
$x^2 + y^2 = 8y + 3x$



$$\frac{72}{4} = \left(x - \frac{3}{2}\right)^2 + (y-4)^2$$

$$x^2 + y^2 = \cancel{8y} \cancel{+3x}$$

$$-3x \quad -8y \quad -8y \quad -3x$$



$$x^2 - 3x + \frac{9}{4} + y^2 - 8y + 16 = 0 + \frac{9}{4} + 16$$

$a = 1$
$b = -3$
$\frac{b}{2} = \frac{-3}{2}$
$\left(\frac{b}{2}\right)^2 = \frac{9}{4}$

$a = 1$
$b = -8$
$\frac{b}{2} = \frac{-8}{2} = -4$
$\left(\frac{b}{2}\right)^2 = 16$

$$\left(x - \frac{3}{2}\right)^2 + (y - 4)^2 = \frac{9}{4} + \frac{64}{4} = \frac{73}{4}$$

center $\left(\frac{3}{2}, 4\right)$

$r = \frac{\sqrt{73}}{2}$

Find the polar coordinates of the point(s) of intersection of the given curves for $0 \le \theta < 2\pi$.

15) $r = 6$, $r = 2 + 4 \sin \theta$

$6 = 2 + 4\sin\theta$
$-2 \quad -2$

$4 = 4\sin\theta$

$1 = \sin\theta$

$\theta = ?$

$$\frac{5x+7}{x^3+2x^2-x-2} = \frac{5x+7}{(x+2)(x+1)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x-1)} = \frac{-1}{x+2} + \frac{-1}{x+1} + \frac{2}{x-1}$$

$$x^2(x+2) - 1(x+2)$$

$$(x+2)(x^2-1)$$

$$(x+2)(x+1)(x-1)$$

$$\frac{5x+7}{(x+2)(x+1)(x-1)} = \frac{A(x+1)(x-1)}{(x+2)(x+1)(x-1)} + \frac{B(x+2)(x-1)}{(x+2)(x+1)(x-1)} + \frac{C(x+2)(x+1)}{(x+2)(x+1)(x-1)}$$

$$5x+7 = A(x^2-1) + B(x^2+x-2) + C(x^2+3x+2)$$

$$5x+7 = Ax^2 - A + Bx^2 + Bx - 2B + Cx^2 + 3Cx + 2C$$

$$0x^2 = Ax^2 + Bx^2 + Cx^2$$

$$5x = Bx + 3Cx$$

$$7 = -A - 2B + 2C$$

$$0 = A + B + C$$

$$5 = 0A + B + 3C$$

$$7 = -A - 2B + 2C$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 5 \\ -1 & -2 & 2 & 7 \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & 3 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 6 & 12 \end{array}\right]$$

$$6C = 12$$

$$C = 2$$

$$B + 3C = 5$$

$$B + 3(2) = 5$$

$$B + 6 = 5$$

$$B = -1$$

$$A + B + C = 0$$

$$A - 1 + 2 = 0$$

$$A = -1$$